SUBJECT: APPLIED MATHEMATICS-I
CODE: MTH301
FACULTY NAME: DR. M.K. SRIVASTAV
PROGRAMME: D.VOC. (INDUSTRIAL ELECTRONICS) &
D.VOC. (MECHANICAL-MANUFACTURING)

Source: NCERT, www.google.com

SYLLABUS

SUBJECT: Applied Mathematics-I
CODE: MTH301
CATEGORY: General Education Component

Credit	Hours	Marks		
4	60	Internal	External	Total
		30	70	100

Unit	Topic	Key Learning
I	Sets, Relations and Functions	• Theory of Sets • Relations • Functions • Polynomials and Graphical Representation
II	Sequence and Series	• Introduction to Sequence and Series • Arithmetic Progression (A.P.) • Geometric Progression (G.P.) • Harmonic Progression (H.P.)
III	Algebra-I	• Partial Fraction • Permutation • Combination • Binomial Theorem
IV	Trigonometry	• Trigonometric Ratio • Compound Angles • Multiple and sub multiple angles • Transformations of products into sums or differences and vice versa
V	Straight Lines	• Cartesian and Polar Coordinate • Different Forms of a Straight Line • General Equation of a Line • Distance of a Point from a Line

Suggested Readings:

1. Mathematics for class XI NCERT.
2. Mathematics for class XII Part I and II NCERT.

Web URLs:

1. www.ncert.nic.in
2. www.nios.ac.in

UNIT-1: SETS, RELATIONS AND FUNCTIONS

Set: A Set is a well-define collection of objects or elements.

Examples:
1. The collection of all boys in your class.

2. The collection of all the months of a year beginning with the letter J = {January, June, July}.

But:
3. The collection of ten most talented writers of India is NOT a Set, because it is not "a well defined collection of objects".

4. A team of eleven best-cricket batsman of the world is NOT a Set, because it is not "a well defined collection of objects".

Note:

- Sets are usually denoted by capital letters A, B, S, X, Y etc.
- The elements of a set are represented by small letters *a*, *b*, *c*, *s*, *x*, *y*, *z*, etc.
- If *a* is an element of a set A, then we say that "*a belongs to* A" and write as "$a \in$ A".
- If *a* is not an element of a set A, then we say that "*a does not belongs to* A" and write as "$a \notin$ A".

Some other examples of sets used in Mathematics:

N : The set of all natural numbers = {1,2,3,…..}
Z : The set of all integers = $\{0, \pm1, \pm2, \pm3, ...\}$
Q : The set of all rational numbers = $\left\{x : x = \frac{a}{b}, a, b \in Z \text{ and } b \notin 0\right\}$
R : The set of real numbers = $(-\infty, +\infty)$

Representation of Set: There are two methods of representing a set:

1. Roster or tabular form.
Example: The set of all even positive integers less than 7 is described in roster form as {2, 4, 6}.

2. Set-builder form.
Example: The set of all vowels in the English alphabet is described in set-builder form as:

{*x* : *x* is a vowel in English alphabet}

Important Examples:
1. Write the set A = {1, 4, 9, 16, 25,.…} in set-builder form.

Answer: We may write the set A = {*x* : *x* is the square of a natural number}

Or, we can write A = $\left\{x : x = n^2, \text{ where } n \in N\right\}$

2. Write the set A = {*x* : *x* is a letter of the word MATHEMATICS} in roster form.

Answer: A = {M, A, T, H, E, I, C, S}

The Empty Set: A set which does not contain any element is called the empty set or the null set.

Symbol: { } or ϕ (phi)

Source: NCERT, www.google.com

Example:

1. Let A = {Students enrolled in Mechatronics and Manufacturing Courses in your class}.
   Then A is the empty set.

   Therefore, $A = \phi$.

2. Let A = {x: 3 < x < 4, x is a natural number}. Then A is the empty set, as there is no natural number between 3 and 4.

   Therefore $A = \phi$.

Finite and Infinite Sets: A set which is empty or consists of a definite number of elements is called finite otherwise, the set is called infinite.

Examples:

1. Let A be the set of the days of the week. Then A is finite.

2. Set of all natural numbers N = {1, 2, 3, 4,….} is Infinite.

Equal Sets: Two sets A and B are said to be equal if they have exactly the same elements and we write A = B.

Otherwise, the sets are said to be unequal and we write $A \neq B$.

Examples: Let A = {1, 2, 3, 4} and B = {3, 1, 4, 2}. Then A = B.

Subsets: A set A is said to be a subset of a set B if every element of A is also an element of B.
We write $A \subset B$ if whenever $a \in$ A, then $a \in$ B.

Therefore, $A \subset B$ if $a \in$ A $\Rightarrow$ $a \in$ B

We read the above statement as "A is a subset of B if a is an element of A implies that a is also an element of B".

Examples: Let A = {1, 2, 3} and B = {1, 2, 3, 4, 5, 6}
Then, A is subset of B. we write as $A \subset B$.

Example: Let A = {a, e, i, o, u} and B = {a, b, c, d, e, f}.
Then A is not a subset of B, also B is not a subset of A.

Example: Let A = {1, 2, 3} and B = {1, 2, 3, 4}
then A is proper subset of B and B is superset of A.

Note:

1. $A \subset B$ and $B \subset A$, $A \Leftrightarrow B$

2. If A is not a subset of B, we write $A \not\subset B$.

3. ϕ is subset of every set.

4. Let A and B be two sets. If A ⊂ B and A ≠ B , then A is called a proper subset of B and B is called superset of A.

5. It is evident that $N \subset Z \subset Q \subset R$.

Closed and open intervals of R:

1. Closed: $\left[a,b\right] = \left\{x : a \leq x \leq b\right\}$

2. Open: $(a,b) = \left\{x : a \leq x \leq b\right\}$

Power Set: The collection of all subsets of a set A is called the power set of A. It is denoted by P(A). In P(A), every element is a set.

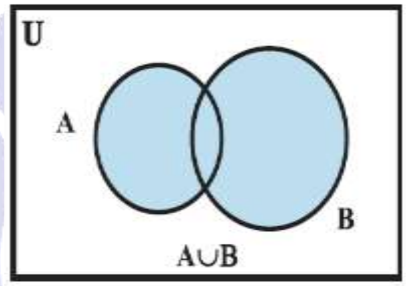
Example: Let A = {1, 2, 3} total 3 elements
Then P(A) = { ϕ , {1,2,3}, {1},{2},{3},{1,2},{1,3},{2,3}}. Total 8 elements.

Note: If number of elements in A = n(A) = m
Then, number of elements in Power set of A = n[P(A)] = 2^m .

Universal Set: A universal set is a set which contains all elements, including itself.

Example: The set of real number R is a universal set.

Union of Sets: Let A and B be any two sets.



The union of A and B is the set which consists of all the elements of A and all the elements of B, the common elements being taken only once.

The symbol "$\cup$" is used to denote the union. We write $A \cup B$ and usually read as 'A union B'.

$$\Rightarrow \quad A \cup B = \{x : x \in A \text{ or } x \in B\}$$

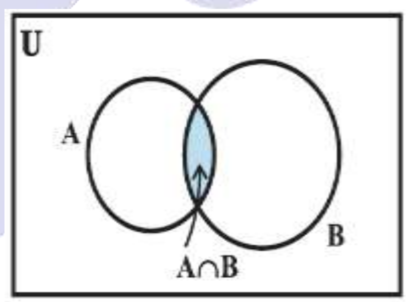
Examples:

1. Let A = {2, 4, 6, 8} and B = {6, 8, 10, 12}
   Then A $\cup$ B = {2, 4, 6, 8, 10, 12} = B $\cup$ A

2. Let A = {a, e, i, o, u} and B = {a, i, u}
   Then A $\cup$ B = {a, e, i, o, u} = A

   This example shows that if B $\subset$ A, then A $\cup$ B = A.

Intersection of Sets: The intersection of sets A and B is the set of all elements which are common to both A and B.



The symbol "$\cap$" is used to denote the intersection. The intersection of two sets A and B is the set of all those elements which belong to both A and B.

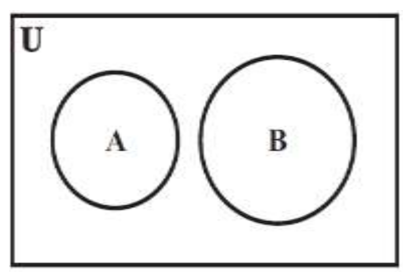
$$\Rightarrow \quad A \cap B = \{x : x \in A \text{ and } x \in B\}.$$

Examples:

1. Let A = {2, 4, 6, 8} and B = {6, 8, 10, 12}.
   Then A $\cap$ B = {6, 8} = B $\cap$ A

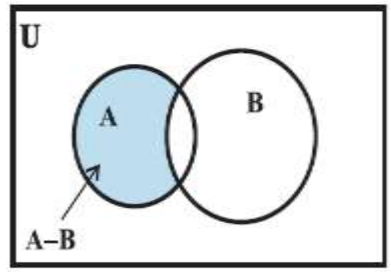
2. A = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} and B = {2, 3, 5, 7}.
   then A $\cap$ B ={ 2, 3, 5, 7 } = B. We note that B $\subset$ A and that A $\cap$ B = B.

Note: Two sets A and B are said to be mutually disjoint sets if A $\cap$ B = ϕ



Example: Let A = {a, b, c} and B = {x, y, z} then
A $\cap$ B = ϕ
Therefore A and B are disjoint sets.

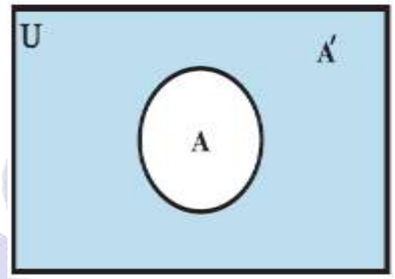
Source: NCERT, www.google.com



<u>Difference of Sets:</u> It is the set of elements which belong to A but not to B.
Symbol: A-B (A minus B)

$\Rightarrow$ A-B $=\{x : x \in$ A and $x \notin$ B$\}$

<u>Example:</u> Let A = {1, 2, 3, 4, 5, 6} and B = {2, 4, 6, 8} then,
A-B = {1, 3, 5}
and B-A = {8}



<u>Complement of a Set:</u> Let U be the universal set and A is a subset of U.

Then the complement of A is the set of all elements of U which are not the elements of A.

it is denoted by A'

$\Rightarrow$ A′ $= \{x : x \in$ U and $x \notin$ A$\}$
Obviously A′ = U-A.

<u>Example:</u> Let U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} and A = {1, 3, 5, 7, 9} then
A′ = {2, 4, 6, 8, 10}
Also (A′)′ = {1, 3, 5, 7, 9} = A

<u>Note:</u>
1. (A′)′ = A
2. U′ = ϕ

<u>Example:</u> Let U = {1, 2, 3, 4, 5, 6}, A = {2, 3} and B = {3, 4, 5}
then A′ = {1, 4, 5, 6} and B′ = {1, 2, 6}
Therefore $A' \cap B' = \{1, 6\}$

Also $A \cup B = \{2, 3, 4, 5\}$, so that $(A \cup B)' = \{1, 6\}$
Therefore $(A \cup B)' = \{1, 6\} = A' \cap B'$

Similarly $A \cap B = \{3\}$, so that $(A \cap B)' = \{1, 2, 4, 5, 6\}$

And $A' \cup B' = \{1, 2, 4, 5, 6\}$
Therefore $(A \cap B)' = \{1, 2, 4, 5, 6\} = A' \cup B'$

<u>Cartesian Products of Sets:</u> Given two non-empty sets A and B.

Then Cartesian product $A \times B$ is set of ordered pairs of elements from A and B.
$\Rightarrow$ $A \times B = \{(a, b) : a \in A, b \in B\}$

If either A or B is the null set, then $A \times B$ will also be empty set $\Rightarrow$ $A \times B = \phi$

<u>Example:</u> Let A = {1, 2, 3, 4}, total elements = 4
and B = $\{a, b, c\}$, total elements = 3

Then,
$A \times B = \{(1, a), (1, b), (1, c), (2, a), (2, b), (2, c), (3, a), (3, b), (3, c), (4, a), (4, b), (4, c)\}$

<u>Note:</u>
1. If number of elements in A = n(A) = m and number of elements in A = n(B) = n,
Number of elements in $A \times B = n(A \times B) = mn$

<u>Relations:</u> A relation R from a non-empty set A to a non-empty set B is a subset of the Cartesian product $A \times B$.
Therefore, $R \subseteq A \times B$.

Source: NCERT, www.google.com

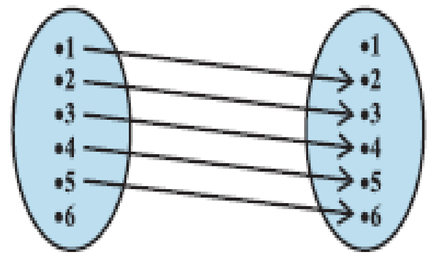
Note:
1. The subset is derived by describing a relationship between the first element and the second element of the ordered pairs in $A \times B$.

2. The second elements are called the image of the first elements.

Example: Let A = {1, 2, 3, 4, 5, 6}. Define a relation R from A to A by
R = {(x, y): $y = x + 1$}
Depict this relation using an arrow diagram.

Solution: By the definition of the relation,

R = {(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)}
The corresponding arrow diagram is described in the figure:



Note: The total number of relations that can be defined from a set A to a set B is the number of possible subsets of $A \times B$.

If $n(A) = m$ and $n(B) = n$, then $n(A \times B) = mn$
and the total number of relations is 2^{mn}.

Example: Let A = {1, 2} then $n(A) = 2$ and B = {3, 4, 5}, then $n(B) = 3$
Therefore,
$A \times B$ = {(1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5)}
Then $n(A \times B) = 6$
and the number of subsets of $A \times B = 2^6$.
Therefore, the number of Relations from A into B will be 2^6.

Function: A relation from a set X to a set Y is said to be a function (f) if every element of set X has one and only one image in set Y.

If f is a function from X to Y and $(x, y) \in f$,
then $f(x) = y$, where y is called the *image* of x under f
and x is called the *preimage* of y under f.

The function (f) from X to Y is denoted by:

$$f : X \rightarrow Y$$

Example: Examine each of the following relations given below and state in each case, giving reasons whether it is a function or not?
(i) R = {(2, 1), (3, 1), (4, 2)}
(ii) R = {(2, 2), (2, 4), (3, 3), (4, 4)}
(iii) R = {(1, 2), (2, 3), (3, 4), (4, 5), (5, 6), (6, 7)}

Answer: (i) Since 2, 3, 4 are the elements of domain of R having their unique images, this relation R is a function.

(ii) Since the same first element 2 corresponds to two different images 2 and 4, this relation is not a function.

(iii) Since every element has one and only one image, this relation is a function.

Polynomial Function: A function $f : \mathrm{R} \to \mathrm{R}$ is said to be polynomial function if for each x in R,

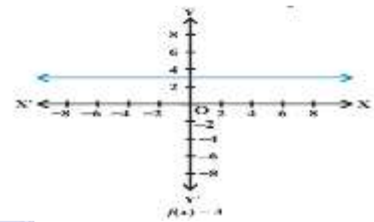
$y = f(x) = a_0 + a_1 x + a_2 x^2 + ... + a_n x^n$ Where, n is a non-negative integer and $a_0, a_1, a_2 ....., a_n \in R$

Example: The functions defined by $f(x) = 8x^2 - 3x + 1$ (Degree = 2) and $g(x) = x^{10} - \sqrt{3}x^4 + x^2 - 13$ (Degree = 10) are some examples of polynomial functions, whereas the function h defined by $h(x) = x^{2/3} + \frac{4}{7}x + 1$ is not polynomial function.

Constant Function: Define the function $f : \mathrm{R} \to \mathrm{R}$ by

$y = f(x) = c,\ x \in \mathrm{R}$

Where, c is a constant and each $x \in$ R.

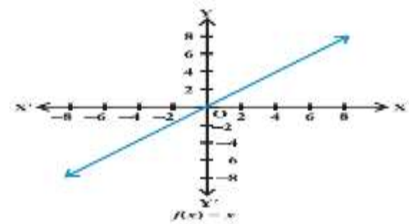


Linear Function: Polynomial functions of degree 1 are called linear function.

$$y = f(x) = a_0 + a_1 x$$

$$a_0,\ a_1 \in \mathrm{R}$$

For example:



Identity Function: Let $f : \mathrm{R} \to \mathrm{R}$ be a real valued function define as:

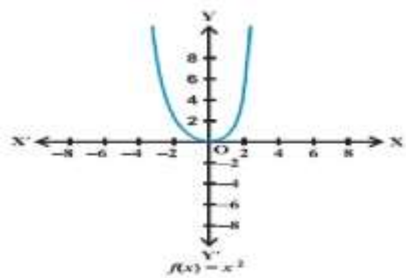
$y = f(x) = x$ for each $x \in$ R.

Quadratic Function: Polynomial functions of degree 2 are called quadratic function.

Therefore function of the form, $y = f(x) = a_0 + a_1 x + a_2 x^2,\ \ a_0,\ a_1, a_2 \in \mathrm{R}$

For example Let $f : \mathrm{R} \to \mathrm{R}$ be a real valued function define as:

(Parabola): $y = f(x) = x^2$ for each $x \in$ R.



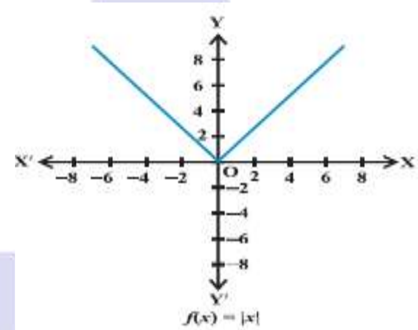
The Modulus Function: The function $f : \mathrm{R} \to \mathrm{R}$ defined by

$f(x) = |x|$ for each $x \in$ R is called modulus function.

For each non-negative value of x, $f(x)$ is equal to x.

But for negative values of x, the value of $f(x)$ is the negative of the value of x.

Therefore, $f(x) = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$



Source: NCERT, www.google.com

UNIT-2: SEQUENCE AND SERIES

Sequence:

We denote the terms of a sequence by $a_1, a_2, a_3, \ldots\ldots, a_n, \ldots\ldots$ The subscript denote the position of the term. The n^{th} term is the number at the n^{th} position of the sequence and is denoted by a_n . The n^{th} term is also called the **general term** of the sequence.

Example: 2, 5, 8,…..(3*n*-1),……

Here $a_1 = 2,\ a_n = 3n - 1$

- A sequence is called **finite sequence** if it contains finite number of terms.
- A sequence is called **infinite**, if it is not a finite sequence.

Fibonacci Sequence: It is a sequence which is generated by the recurrence relation given by:

$$a_1 = a_2 = 1$$
$$a_n = a_{n-2} + a_{n-1},\ n > 2$$

i.e. 1, 1, 2, 3, 5, 8, 13,…………

Series: Let $a_1, a_2, a_3, \ldots\ldots, a_n$, be a give sequence.

Then $S_n = a_1 + a_2 + a_3 + \ldots\ldots + a_n + \ldots$ is called the **series associated with the given sequence.** The series is finite or infinite according to the given sequence is finite or infinite.

Arithmetic Progression (AP):

Arithmetic progression (AP) or arithmetic sequence is a sequence of numbers in which each term after the first is obtained by adding a constant, *d* to the preceding term. The constant *d* is called **common difference**.

An AP is given as: $a, a+d, a+2d, a+3d, \ldots\ldots\ldots$

Here, a = the first term, d = the common difference

Example: 1, 3, 5, 7,…… is an AP with a =1 and d = 2

*n*th term of an arithmetic progression: $T_n = a + (n-1)d$

a = the first term, d = the common difference

Example: To find the 20th term of the AP 1, 3, 5, 7…….

Answer: Here, The first term: a = 1, The common difference: d = 2 , n = 20

Then the n^{th} term :

$$T_n = a + (n-1)d$$
$$T_{20} = 1 + (20-1)2$$
$$= 39$$

Source: NCERT, www.google.com

<u>Sum of first *n* terms in an Arithmetic Progression:</u>

Sum of first *n* terms in an AP is given by: $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

Here, a = the first term, d = the common difference

Example: To find $4+7+10+13+\ldots\ldots$ up to 20 terms.

Answer: Here, The first term: a = 4, The common difference: d = 3

Therefore,

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$
$$S_{20} = \frac{20}{2}\left[2\times 4 + (20-1)\times 3\right]$$
$$= 10\left[8 + 19\times 3\right]$$
$$= 650$$

<u>Geometric Progression (GP):</u>

Geometric Progression (GP) or Geometric Sequence is sequence of non-zero numbers in which the ratio of any tern and its preceding term is always constant.

A Geometric Progression (GP) is given by $a, ar, ar^2, ar^3, \ldots$

Where a = the first term, r = the common ratio

Example: 1, 3, 9, 27,….. is a geometric progression (GP) with a = 1 and r = 3

<u>*n*th term of a Geometric Progression</u>: $T_n = ar^{n-1}$

a = the first term, r = the common ratio

Example: To find the 10th term in the series $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots\ldots$

Answer: Here, the first term: $a = \frac{1}{2}$, the common ratio = $r = \frac{\left(\frac{1}{4}\right)}{\left(\frac{1}{2}\right)} = \frac{1}{2}$

Then, for 10th term: $T_n = ar^{n-1}$

$$T_{10} = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^{10-1} = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^{9} = \left(\frac{1}{2}\right)^{10} = \frac{1}{1024}$$

Sum of first *n* terms in a Geometric Progression:

$$S_n = \begin{cases} \dfrac{a\left(r^n - 1\right)}{r-1}, & (r > 1) \\ \dfrac{a\left(1 - r^n\right)}{1-r}, & (r < 1) \end{cases}$$

Where, a = first term,

r = common ratio,

n = number of terms

Example: To find $4 + 12 + 36 + \ldots\ldots$ up to 6 terms

Answer: Here, a = 4, $r = \frac{12}{4} = \frac{36}{12} = 3,\ n = 6$

Therefore, r > 1, hence

$$S_n = \frac{a\left(r^n - 1\right)}{r-1}$$

$$S_6 = \frac{4\left(3^6 - 1\right)}{3-1} = \frac{4\left(729 - 1\right)}{2} = \frac{4 \times 728}{2} = 1456$$

Sum of an infinite Geometric Progression:

$$S_\infty = \frac{a}{1-r}, \quad -1 < r < 1$$

Where, a = the first term, r = the common ratio

Example: To find $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots..\infty$

Answer: a =1, $r = \frac{\left(\frac{1}{2}\right)}{1} = \frac{1}{2}$

Here, $-1 < r < 1$, Hence: $S_\infty = \frac{a}{1-r} = \frac{1}{\left(1 - \frac{1}{2}\right)} = \frac{1}{\left(\frac{1}{2}\right)} = 2$

Harmonic Progression (HP): Non-Zero numbers $T_1, T_2, T_3, T_4, \ldots.T_n$ are in Harmonic Progression if $\frac{1}{T_1}, \frac{1}{T_2}, \frac{1}{T_3}, \frac{1}{T_4}, \ldots\ldots.\frac{1}{T_n}$ are in Arithmetic Progression.

Source: NCERT, www.google.com

Example: $\frac{1}{3},\frac{1}{5},\frac{1}{7},\frac{1}{9},.....$ is a harmonic progression

Arithmetic Mean (AM), Geometric Mean (GM) and Harmonic Mean (HM):

If a, b are two numbers, then AM = $\frac{a+b}{2}$

GM = $\sqrt{ab}$ (both a, b are of same sign)

HM = $\frac{2ab}{a+b}$

Relation Between AM, GM, HM 1. $GM = \sqrt{AM * HM}$

2. $AM > GM > HM$

Important Formula 1. $1+2+3+.....+n = \sum n = \frac{n(n+1)}{2}$

2. $1^2+2^2+3^2+.....+n^2 = \sum n^2 = \frac{n(n+1)(2n+1)}{6}$

3. $1^3+2^3+3^3+.....+n^3 = \sum n^3 = \frac{n^2(n+1)^2}{4}$

UNIT-3: ALGEBRA-I

Partial Fraction:

An algebraic fraction is a fraction in which the numerator and denominator are both polynomial expressions.

Example:

$$\left.\begin{array}{ll} 1. & \dfrac{3x}{x^2+2x+1} \\ 2. & \dfrac{1-x^3}{x^4-2x^2+1} \end{array}\right\} \text{Proper fraction (The numerator is a polynomial of lower degree than the denominator)}$$

$$\left.\begin{array}{ll} 3. & \dfrac{x^4+3x^3+x}{x^2-x+7} \\ 4. & \dfrac{x+1}{1-x} \end{array}\right\} \text{Improper fraction (The numerator is a polynomial of higher degree than the denominator)}$$

Expressing a fraction as the sum of its partial fractions:

To express a single algebraic fraction into the sum of two or more single algebraic fraction is called **Partial fraction resolution** and these algebraic fraction are called **Partial fractions.**
The method for computing partial fraction decompositions applies to all partial functions with one qualification:

- The degree of the numerator must be less than the degree of the denominator (i.e. for Proper fraction only)

Rule: **1.** For a linear term $ax+b$ in the denominator, we get a partial fraction as: $\dfrac{A}{ax+b}$

2. For a repeated linear term in the denominator, such as $(ax+b)^3$, we get partial fractions as:

$$\frac{A}{ax+b}+\frac{B}{(ax+b)^2}+\frac{C}{(ax+b)^3}$$

3. For a quadratic term ax^2+bx+c in the denominator, we get we partial fractions as: $\dfrac{Ax+B}{ax^2+bx+c}$

4. For a repeated quadratic term in the denominator, such as $(ax^2+bx+c)^2$, we get partial fractions as:

$$\frac{Ax+B}{ax^2+bx+c}+\frac{Cx+D}{(ax^2+bx+c)^2}$$

Example:

$$\text{I. } \frac{x-1}{(x+2)(x-7)}=\frac{A}{(x+2)}+\frac{B}{(x-7)}$$

$$\text{II. } \frac{2x^2+1}{x^3-x^2-8x+12}=\frac{2x^2+1}{(x-2)^2(x+3)}=\frac{A}{(x-2)}+\frac{B}{(x-2)^2}+\frac{C}{(x+3)^2}$$

$$\text{III. } \frac{x^2+1}{(x^2+x+2)(x+7)}=\frac{Ax+B}{(x^2+x+2)}+\frac{C}{(x+7)}$$

$$\text{IV. } \frac{x^2+1}{(x-1)(x+2)(x^2+2x+5)^2}=\frac{A}{(x-1)}+\frac{B}{(x+2)}+\frac{Cx+D}{(x^2+2x+5)}+\frac{Ex+F}{(x^2+2x+5)^2}$$

Computing the coefficients: First we determine the right form for the partial fraction decomposition of an algebraic fraction, then to compute the unknown coefficients *A*, *B*, *C*,... we use two methods for this purpose. We will now look at both methods for the decomposition of:

$$\frac{2x-1}{(x+2)^2(x-3)}$$

Applying above rules:

$$\frac{2x-1}{(x+2)^2(x-3)} = \frac{A}{(x+2)} + \frac{B}{(x+2)^2} + \frac{C}{(x-3)}$$

$$\Rightarrow \quad \frac{2x-1}{(x+2)^2(x-3)} = \frac{A(x+2)(x-3)+B(x-3)+C(x+2)^2}{(x+2)^2(x-3)}$$

$$\Rightarrow \quad 2x-1 = A(x+2)(x-3)+B(x-3)+C(x+2)^2 \qquad ......(1)$$

- In the ***first method***, we substitute different values for x in to the equation no. (1) and deduce the values of A, B, C.

So putting $x=3$ in equation (1) we have:

$$2\times 3-1 = A(3+2)(3-3)+B(3-3)+C(3+2)^2$$

$$\Rightarrow \quad 5 = 25C$$

$$\Rightarrow \quad C = \frac{1}{5}$$

Putting $x=-2$ in equation (1) we have:

$$2\times(-2)-1 = A(-2+2)(-2-3)+B(-2-3)+C(-2+2)^2$$

$$\Rightarrow \quad -5 = -5B$$

$$\Rightarrow \quad B = 1$$

Putting $x=0$ in equation (1) we have:

$$2\times 0-1 = A(0+2)(0-3)+B(0-3)+C(0+2)^2$$

$$\Rightarrow \quad -1 = -6A-3B+4C$$

$$\Rightarrow -1 = -6A-3(1)+4\left(\frac{1}{5}\right) \Rightarrow \quad A = -\frac{1}{5}$$

Therefore: $$\frac{2x-1}{(x+2)^2(x-3)} = \frac{\left(\frac{-1}{5}\right)}{(x+2)} + \frac{1}{(x+2)^2} + \frac{\left(\frac{1}{5}\right)}{(x-3)}$$

Permutation:

Fundamental Principle of Counting: If an event can occur in *m* different ways, following which another event can occur in *n* different ways, then the total number of occurrence of the events in the given order is $m \times n$.

Example: Find the number of 5 letter words, with or without meaning, which can be formed out of the letters of the word KUMAR, where the repetition of the letters is not allowed.
Solution: There are as many words as there are ways of filling in 5 vacant places by the 5 letters, keeping in mind that the repetition is not allowed.

- The first place can be filled in 5 different ways by anyone of the 5 letters K, U, M, A, R.
- Following which, the second place can be filled in by anyone of the remaining 4 letters in 4 different ways
- Following which the third place can be filled in 3 different ways
- Following which, the fourth place can be filled in 2 ways.
- Following which, the fourth place can be filled in 1 way

Thus, the number of ways in which the 5 places can be filled, by the multiplication principle, is $5 \times 4 \times 3 \times 2 \times 1 = 120$.

Note: If the repetition of the letters was allowed, how many words can be formed? One can easily understand that each of the 5 vacant places can be filled in succession in 4 different ways. Hence, the required number of words = $5 \times 5 \times 5 \times 5 = 3125$.

Example: Given 4 flags of different colours, how many different signals can be generated, if a signal requires the use of 2 flags one below the other?
Solution: There will be as many signals as there are ways of filling in 2 vacant places in succession by the 4 flags of different colours. The upper vacant place can be filled in 4 different ways by anyone of the 4 flags; following which, the lower vacant place can be filled in 3 different ways by anyone of the remaining 3 different flags. Hence, by the multiplication principle, the required number of signals = $4 \times 3 = 12$.

Example: How many 2 digits odd numbers can be formed from the digits 1, 2, 3, 4, 5 if the digits can be repeated?
Solution: There will be as many ways as there are ways of filling 2 vacant places in succession by the five given digits. Here, in this case, we start filling in unit's place, because the options for this place are 1, 3 and 5 only and this can be done in 3 ways; following which the ten's place can be filled by any of the 5 digits in 5 different ways as the digits can be repeated. Therefore, by the multiplication principle, the required number of two digits even numbers are $3 \times 5 = 15$.

Factorial of Number: The factorial of a natural number *n* is denoted by $n!$ and it is the product of first *n* natural numbers.

Therefore,
$$n! = 1 \times 2 \times 3 \times \ldots\ldots \times (n-1) \times n$$
$$Or,\ n! = n \times (n-1) \times \ldots\ldots \times 3 \times 2 \times 1$$

Example:
$$0! = 1$$
$$1! = 1$$
$$2! = 2 \times 1 = 2$$
$$3! = 3 \times 2 \times 1 = 6$$
$$4! = 4 \times 3 \times 2 \times 1 = 24$$
Clearly, $n! = n \times (n-1)!$

Example: Calculate the value of $\frac{8!}{5!}$

Solution: $\frac{8!}{5!} = \frac{8 \times 7 \times 6 \times 5!}{5!} = 336$

Source: NCERT, www.google.com

Example: If $\frac{1}{5!}+\frac{1}{6!}=\frac{n}{7!}$, find *n*.

Solution: We have,

$$\frac{1}{5!}+\frac{1}{6!}=\frac{n}{7!} \quad \Rightarrow \frac{1}{5!}+\frac{1}{6\times 5!}=\frac{n}{7\times 6\times 5!} \quad \Rightarrow 1+\frac{1}{6}=\frac{n}{7\times 6} \quad \Rightarrow \frac{7}{6}=\frac{n}{7\times 6}$$

$$\Rightarrow n=49$$

Permutation: A permutation is an arrangement in a definite order of a number of objects taken some or all at a time.

Case1. Permutations when all the objects are distinct:

The number of permutations of *n* different objects taken *r* at a time, where $0<r\le n$ and the objects do not repeat is *n* (*n* – 1) (*n* – 2) . . . (*n* – *r* + 1), which is denoted by ${}^{n}P_{r}$.

$${}^{n}P_{r}=\frac{n!}{(n-r)!},\ 0\le r\le n$$

Example: How many 5-digit numbers can be formed by using the digits 1 to 9 if repetition of digits is not allowed?

Solution: Here order matters for example 12345 and 13245 are two different numbers. Therefore, there will be as many 5 digit numbers as there are permutations of 9 different digits taken 5 at a time. Therefore, the required 5 digit numbers = ${}^{9}P_{4}=\frac{9!}{(9-5)!}=\frac{9!}{4!}=9\times 8\times 7\times 6\times 5=15120$

Case2: The number permutations of *n* different objects taken *r* at a time, where repetition is allowed, is n^{r}.

Case3. Permutations when all the objects are not distinct:

The number of permutations of *n* objects, where p_1 objects are of one kind, p_2 are of second kind, ..., p_k are of k^{th} kind and the rest, if any, are of different kind is $\frac{n!}{p_1!\,p_2!....p_k!}$

Example: Find the number of permutations of the letters of the word ALLAHABAD.

Solution: Here, there are 9 objects (letters) of which there are 4A's, 2 L's and rest are all different.

Therefore, the required number of arrangements = $\frac{9!}{4!2!}=\frac{9\times 8\times 7\times 6\times 5\times 4!}{4!2!}=7560$

Example: How many numbers lying between 100 and 1000 can be formed with the digits 0, 1, 2, 3, 4, 5, if the repetition of the digits is not allowed?

Solution: Every number between 100 and 1000 is a 3-digit number. We, first, have to count the permutations of 6 digits taken 3 at a time. This number would be ${}^{6}P_{3}$. But, these permutations will include those also where 0 is at the 100's place. For example, 092, 042, . . ., etc are such numbers which are actually 2-digit numbers and hence the number of such numbers has to be subtracted from ${}^{6}P_{3}$ to get the required number. To get the number of such numbers, we fix 0 at the 100's place and rearrange the remaining 5 digits taking 2 at a time. This number is ${}^{5}P_{2}$. So

The required number ${}^{6}P_{3}-{}^{5}P_{2}=\frac{6!}{3!}-\frac{5!}{3!}=6\times 5\times 4-5\times 4=100$

Source: NCERT, www.google.com

Example: Find the number of different 8-letter arrangements that can be made from the letters of the word DAUGHTER so that (i) all vowels occur together
(ii) all vowels do not occur together.

Solution: (i) There are 8 different letters in the word DAUGHTER, in which there are 3 vowels, namely, A, U and E. Since the vowels have to occur together, we can for the time being, assume them as a single object (AUE). This single object together with 5 remaining letters (objects) will be counted as 6 objects. Then we count permutations of these 6 objects taken all at a time.

This number would be ${}^{6}P_{6} = \frac{6!}{(6-6)!} = 6!$. Corresponding to each of these permutations, we shall have 3! permutations of the three vowels A, U, E taken all at a time.
Hence, by the required number of permutations = 6! × 3! = 4320.

(ii) If we have to count those permutations in which all vowels are never together, we first have to find all possible arrangements of 8 letters taken all at a time, which can be done in 8! ways. Then, we have to subtract from this number, the number of permutations in which the vowels are always together.
Therefore, the required number = 8! – 6! × 3! = 6! (7×8 – 6)
= 2 × 6! (28 – 3)
= 50 × 6! = 50 × 720 = 36000

Example: Find the value of n such that ${}^{n}P_{5} = 42\,{}^{n}P_{3},\ n > 4$

Solution: Given that ${}^{n}P_{5} = 42\,{}^{n}P_{3}$

$$\Rightarrow \frac{n!}{(n-5)!} = 42\frac{n!}{(n-3)!}$$

$$\Rightarrow n\,(n-1)\,(n-2)\,(n-3)\,(n-4) = 42\,n(n-1)\,(n-2)$$

Since $n > 4$ then $n\,(n-1)\,(n-2) \neq 0$

Therefore, by dividing both sides by $n(n-1)\,(n-2)$, we get

$$\Rightarrow n^2 - 7n - 30 = 0$$

$$\Rightarrow n^2 - 10n + 3n - 30 = 0$$

$$\Rightarrow (n-10)(n+3) = 0$$

$$\Rightarrow n - 10 = 0 \text{ or } n + 3 = 0$$

$$\Rightarrow n = 10 \text{ or } n = -3$$

As n cannot be negative, so $n = 10$

Combinations: A collection of things, in which the order does not matter.

Example: Suppose we have a set of three letters: A, B, and C. Then how many ways we can select 2 letters from this set. Each possible selection would be an example of a combination. The complete list of possible selections would be: AB, BC, CA = 03.

Note:

$$(i)\,{}^{n}C_{r} = \frac{{}^{n}P_{r}}{r!} = \frac{n!}{r!(n-r)!},\ 0 \le r \le n$$

$$(ii)\,{}^{n}C_{r} + {}^{n}C_{r-1} = {}^{n+1}C_{r}$$

Example: If ${}^nC_5 = {}^nC_6$, find the value of *n*.

Solution: we have

$$
\begin{aligned}
&{}^nC_5 = {}^nC_6 \\
&\Rightarrow \frac{n!}{5!(n-5)!} = \frac{n!}{6!(n-6)!} \\
&\Rightarrow \frac{1}{5!(n-5)(n-6)!} = \frac{1}{6\times 5!(n-6)!} \\
&\Rightarrow \frac{1}{(n-5)} = \frac{1}{6} \\
&\Rightarrow n = 11
\end{aligned}
$$

Example: A committee of 3 persons is to be constituted from a group of 2 men and 3 women. In how many ways can this be done? How many of these committees would consist of 1 man and 2 women?

Solution: Here, order does not matter. Therefore, we need to count combinations. There will be as many committees as there are combinations of 5 different persons taken 3 at a time.

Hence, the required number of ways = ${}^5C_3 = \frac{5!}{3!2!} = 10$

Now, 1 man can be selected from 2 men in ${}^2C_1$ ways and 2 women can be selected from 3 women in ${}^3C_2$ ways.

Therefore, the required number of committees = ${}^2C_1 \times {}^3C_2 = \frac{2!}{1!1!} \times \frac{3!}{2!1!} = 6$

Example: What is the number of ways of choosing 4 cards from a pack of 52 playing cards? In how many of these
(i) four cards are of the same suit,
(ii) four cards belong to four different suits,
(iii) are face cards,
(iv) two are red cards and two are black cards,
(v) cards are of the same colour?

Solution: There will be as many ways of choosing 4 cards from 52 cards as there are combinations of 52 different things, taken 4 at a time.
Therefore, The required number of ways

$$= {}^{52}C_4 = \frac{52!}{4!48!} = \frac{52\times 51\times 50\times 49\times 48!}{4\times 3\times 2\times 48!} = 270775$$

(i) There are four suits: diamond, club, spade, heart and there are 13 cards of each suit. Therefore, there are ${}^{13}C_4$ ways of choosing 4 diamonds. Similarly, there are ${}^{13}C_4$ ways of choosing 4 clubs, ${}^{13}C_4$ ways of choosing 4 spades and ${}^{13}C_4$ ways of choosing 4 hearts. Therefore, The required number of ways =

$$
\begin{aligned}
&{}^{13}C_4 + {}^{13}C_4 + {}^{13}C_4 + {}^{13}C_4 \\
&= 4\times {}^{13}C_4 \\
&= 4\times \frac{13!}{4!9!} = 2860
\end{aligned}
$$

(ii) There are13 cards in each suit. Therefore, there are 13C1 ways of choosing 1 card from 13 cards of diamond, ${}^{13}C_1$ ways of choosing 1 card from 13 cards of hearts, ${}^{13}C_1$ ways of choosing 1card from 13 cards of clubs, ${}^{13}C_1$ ways of choosing 1 card from 13 cards of spades. Hence, by multiplication principle, the required number of ways

$$= {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1$$
$$= 13 \times 13 \times 13 \times 13$$
$$= 13^4$$

(iii) There are 12 face cards and 4 are to be selected out of these 12 cards. This can be done in ${}^{12}C_4$ ways. Therefore, the required number of ways $= \frac{12!}{4!8!} = 495$

(iv) There are 26 red cards and 26 black cards. Therefore, the required number of ways

$$= {}^{26}C_2 \times {}^{26}C_2$$
$$= \left(\frac{26!}{2!24!}\right)^2 = (325)^2 = 105625$$

(v) 4 red cards can be selected out of 26 red cards on ${}^{26}C_4$ ways. 4 black cards can be selected out of 26 black cards in ${}^{26}C_4$ ways. Therefore, the required number of ways

$$= {}^{26}C_4 + {}^{26}C_4$$
$$= 2 \times {}^{26}C_4$$
$$= 2 \times \frac{26!}{4!22!} = 29900$$

Binomial theorem for any positive integer *n*:

$$(x+y)^n = {}^nC_0 x^n + {}^nC_1 x^{n-1} y + {}^nC_2 x^{n-2} y^2 + \dots\dots + {}^nC_{n-1} x y^{n-1} + {}^nC_n y^n$$

Note:

1. The above expression can also be written as: $(x+y)^n = \sum_{r=0}^{n} {}^nC_r x^{n-r} y^r$
2. The coefficients ${}^nC_r$ occurring in the binomial theorem are known as binomial coefficients.
3. There are (n +1) terms in the expansion of $(x+y)^n$, *i.e.*, one more than the index (n).
4. Similarly $(x-y)^n = {}^nC_0 x^n - {}^nC_1 x^{n-1} y + {}^nC_2 x^{n-2} y^2 + \dots\dots + (-1)^n\, {}^nC_n y^n$

Example: Expand $(x^2 + \frac{3}{x})^4, x \neq 0$

Solution: Formula $(x+y)^n = {}^nC_0 x^n + {}^nC_1 x^{n-1} y + {}^nC_2 x^{n-2} y^2 + \dots\dots + {}^nC_{n-1} x y^{n-1} + {}^nC_n y^n$

Therefore:

$$(x^2 + \frac{3}{x})^4 = {}^4C_0 (x^2)^4 + {}^4C_1 (x^2)^3 \left(\frac{3}{x}\right) + {}^4C_2 (x^2)^2 \left(\frac{3}{x}\right)^2 + {}^4C_3 (x^2) \left(\frac{3}{x}\right)^3 + {}^4C_4 \left(\frac{3}{x}\right)^4$$
$$= x^8 + 4x^6.\left(\frac{3}{x}\right) + 6x^4.\left(\frac{9}{x^2}\right) + 4x^2.\left(\frac{27}{x^3}\right) + \left(\frac{81}{x^4}\right)$$
$$= x^8 + 12x^5 + 54x^2 + \frac{108}{x} + \frac{81}{x^4}$$

Example: Compute $(98)^5$

Solution: Write 98 = 100-2

Therefore $(98)^5 = (100-2)^5$

$$= {}^5C_0(100)^5 - {}^5C_1(100)^4.(2) + {}^5C_2(100)^3.(2)^2 - {}^5C_3(100)^2.(2)^3 + {}^5C_4(100).(2)^4 - {}^5C_5(2)^5$$

$$= 10000000000 - 5\times 100000000\times 2 + 10\times 1000000\times 4 - 10\times 10000\times 8 + 5\times 100\times 16 - 32$$

$$= 10040008000 - 1000800032 = 9039207968$$

General Term:

General term in the expansion of $(x+y)^n$ is given by

$$T_{r+1} = {}^nC_r x^{n-r} y^r$$

Middle Terms: **(1)** If n is even, then the middle term is $\left(\frac{n}{2}+1\right)^{\text{th}}$ term.

(2) If n is odd, then the middle terms are $\left(\frac{n+1}{2}\right)^{\text{th}}$ term and $\left(\frac{n+1}{2}+1\right)^{\text{th}}$ term.

Example: In the expansion of $\left(x+\frac{1}{x}\right)^{2n}$, where $x \neq 0$,

The middle term is $\left(\frac{2n+1+1}{2}\right)^{\text{th}} = (n+1)^{th}$ term. ($2n$ is even)

It is given by $T_{n+1} = {}^{2n}C_n x^n \left(\frac{1}{x}\right)^n = {}^{2n}C_n$ (constant)

Example: Find a if the 17th and 18th term of the expansion $(2+a)^{50}$ are equal.

Solution: The $(r+1)^{th}$ term of the expansion $(x+y)^n$ is

$$T_{r+1} = {}^nC_r x^{n-r} y^r$$

For the 17th term, we have $r + 1 = 17$ so $r = 16$

$\Rightarrow T_{17} = T_{16+1} = {}^{50}C_{16}(2)^{50-16}(a)^{16} = {}^{50}C_{16}(2)^{34}(a)^{16}$

similarly, $T_{18} = T_{17+1} = {}^{50}C_{17}(2)^{50-17}(a)^{17} = {}^{50}C_{17}(2)^{33}(a)^{17}$

now $T_{17} = T_{18}$

so ${}^{50}C_{16}(2)^{34}(a)^{16} = {}^{50}C_{17}(2)^{33}(a)^{17}$

Therefore $\dfrac{{}^{50}C_{16}(2)^{34}(a)^{16}}{{}^{50}C_{17}(2)^{33}(a)^{17}} = \dfrac{a^{17}}{a^{16}}$

so, $a = \dfrac{{}^{50}C_{16}\times 2}{{}^{50}C_{17}} = \dfrac{50!}{16!34!}\times\dfrac{17!\,33!}{50!}\times 2 = 1$

Example: Find the coefficient of x^6y^3 in the expansion of $(x+2y)^9$.

Solution: Suppose x^6y^3 occurs in the $(r+1)^{th}$ term of the expansion $(x+2y)^9$

Now $T_{r+1} = {}^9C_r x^{9-r}(2y)^r = {}^9C_r 2^r x^{9-r} y^r$

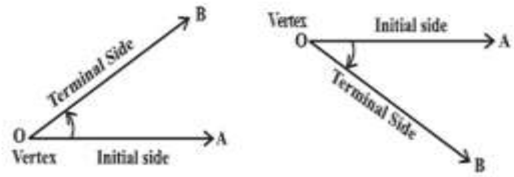
Comparing the indices of x as well as in x^6y^3 and in T_{r+1}, we get $r=3$

Thus the coefficient of x^6y^3 is: ${}^9C_3 2^3 = \dfrac{9!}{3!6!}.2^3 = \dfrac{9.8.7}{3.2}.2^3 = 672$

Source: NCERT, www.google.com

UNIT-4: TRIGONOMETRIC FUNCTIONS

Angles: Angle is a measure of rotation of a ray about its initial point. The original ray is called the *initial side* and the final position of the ray after rotation is called the *terminal side* of the angle. The point of rotation is called the *vertex*. If the direction of rotation is anticlockwise, the angle is said to be positive and if the direction of rotation is clockwise, then the angle is *negative.*



(i)Positive angle (ii) Negative angle

Degree Measure: If a rotation from the initial side is $\left(\frac{1}{360}\right)^{th}$ of a revolution, the angle is said to have a measure of one degree, written as 1^0. A degree is divided into 60 minutes, and a minute is divided into 60 seconds. $\left(\frac{1}{60}\right)^{th}$ of a degree is called a *minute*, written as $1'$, and one sixtieth of a minute is called a *second*, written as $1''$.

Thus, $1^0 = 60',\ 1' = 60'',$

- Radian measure $= \frac{\pi}{180} \times$ Degree measure
- Degree measure $= \frac{180}{\pi} \times$ Radian measure

Radian Measure: Another unit for measurement of an angle is called the *radian* measure.

$$2\pi \text{ radian} = 360^0 \text{ or } \pi \text{ radian} = 180^0.$$

$$1 \text{ radian} = \frac{180^0}{\pi} = 57^0\,16' \text{ appoximately.}$$

$$\text{Also, } 1^0 = \frac{\pi}{180} \text{ radian} = \frac{\left(\frac{22}{7}\right)}{180} \text{ radian} = 0.01746 \text{ radian appoximately.}$$

Example: The relation between degree measures and radian measure of some common angles are given in the following table:

Degree	30^0	45^0	60^0	90^0	180^0	270^0	360^0
Radian	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	π	$3\pi/2$	2π

Example: Convert $40^0 20'$ into radian measure.

Solution: We know that $180^0 = \pi$ radian

Hence, $$40^0 20' = 40\frac{1}{3} \text{degree} = \frac{\pi}{180} \times \frac{121}{3} \text{radian} = \frac{121\pi}{540} \text{radian}$$

Example: Convert 6 radians into degree measure.

Solution: we know that π radian = 180^0

$$6 \text{ radians} = \frac{180}{\pi} \times 6 \text{ degree} = \frac{1080 \times 7}{22} \text{degree}$$

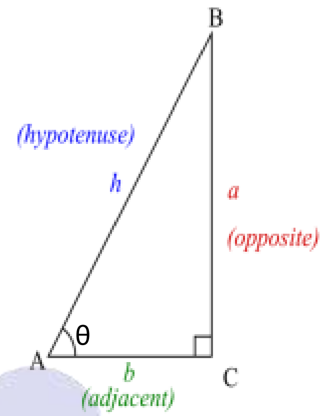
$$= 343\frac{7}{11} \text{degree} = 343^0 + \frac{7 \times 60}{11} \text{minute}$$

$$= 343^0 + 38' + \frac{2}{11} \text{minute} = 343^0 + 38' + 10.9''$$

$$= 343^0 38' 11'' \text{ approximetly}$$

Source: NCERT, www.google.com

Trigonometry Ratio:



$$\sin\theta=\frac{\text{opposite}}{\text{hypotenuse}},\qquad \cos\theta=\frac{\text{adjacent}}{\text{hypotenuse}},\qquad \tan\theta=\frac{\sin\theta}{\cos\theta}=\frac{\text{opposite}}{\text{adjacent}}$$

$$\text{cosec}\theta=\frac{1}{\sin\theta}=\frac{\text{hypotenuse}}{\text{opposite}},\ \sec\theta=\frac{1}{\cos\theta}=\frac{\text{hypotenuse}}{\text{adjacent}},\ \cot\theta=\frac{1}{\tan\theta}=\frac{\text{adjacent}}{\text{opposite}}$$

- $\sin^2\theta+\cos^2\theta=1$
- $1+\tan^2\theta=\sec^2\theta$
- $1+\cot^2\theta=\text{cosec}^2\theta$
- $\sin(-\theta)=-\sin\theta,\quad \cos(-\theta)=\cos\theta$
- $\tan(-\theta)=-\tan\theta,\ \cot(-\theta)=-\cot\theta$
- $\sec(-\theta)=\sec\theta,\quad \text{cosec}(-\theta)=-\text{cosec}\theta$

Review of ratio of some standard angles:

Angle ($\boldsymbol{\theta}$)	$\mathbf{0^0}$	$\mathbf{30^0}$	$\mathbf{45^0}$	$\mathbf{60^0}$	$\mathbf{90^0}$
sinθ	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
cosθ	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
tanθ	0	$1/\sqrt{3}$	1	$\sqrt{3}$	∞
cosecθ	∞	2	$\sqrt{2}$	$2/\sqrt{3}$	1
secθ	1	$2/\sqrt{3}$	$\sqrt{2}$	2	∞
cotθ	∞	$\sqrt{3}$	1	$1/\sqrt{3}$	0

- $\sin\left(\frac{\pi}{2}-\theta\right)=\cos\theta,\quad \cos\left(\frac{\pi}{2}-\theta\right)=\sin\theta$
- $\tan\left(\frac{\pi}{2}-\theta\right)=\cot\theta,\quad \cot\left(\frac{\pi}{2}-\theta\right)=\tan\theta$
- $\sec\left(\frac{\pi}{2}-\theta\right)=\text{cosec}\theta,\ \text{cosec}\left(\frac{\pi}{2}-\theta\right)=\sec\theta$
- $\sin(\pi-\theta)=\sin\theta,\quad \cos(\pi-\theta)=-\cos\theta$
- $\tan(\pi-\theta)=-\tan\theta,\ \cot(\pi-\theta)=-\cot\theta$
- $\sec(\pi-\theta)=-\sec\theta,\ \text{co}\sec(\pi-\theta)=\text{co}\sec\theta$

- $\sin(\pi+\theta)=-\sin\theta,\quad \cos(\pi+\theta)=-\cos\theta$
- $\tan(\pi+\theta)=\tan\theta,\quad \cot(\pi+\theta)=\cot\theta$
- $\sec(\pi+\theta)=-\sec\theta,\ \text{co}\sec(\pi+\theta)=-\text{co}\sec\theta$
- $\sin(2\pi-\theta)=-\sin\theta,\ \cos(2\pi-\theta)=\cos\theta$
- $\tan(2\pi-\theta)=-\tan\theta,\cot(2\pi-\theta)=-\cot\theta$
- $\sec(2\pi-\theta)=\sec\theta,\quad \text{co}\sec(2\pi-\theta)=-\text{co}\sec\theta$

Source: NCERT, www.google.com

Note: The signs of other trigonometric functions in different quadrants are as follows:

Quadrant	Value
I	**A** (All Positive)
II	**S** (only sinθ and cosecθ are positive and others are negative)
III	**T** (only tanθ and cotθ are positive and others are negative)
IV	**C** (only cosθ and secθ are positive and others are negative)

Compound Angles:

- $\sin(x+y) = \sin x \cos y + \cos x \sin y$
- $\sin(x-y) = \sin x \cos y - \cos x \sin y$
- $\cos(x+y) = \cos x \cos y - \sin x \sin y$
- $\cos(x-y) = \cos x \cos y + \sin x \sin y$
- $\tan(x+y) = \dfrac{\tan x + \tan y}{1-\tan x \tan y}$, $\tan(x-y) = \dfrac{\tan x - \tan y}{1+\tan x \tan y}$
- $\cot(x+y) = \dfrac{\cot x \cot y - 1}{\cot y + \cot x}$, $\cot(x-y) = \dfrac{\cot x \cot y + 1}{\cot y - \cot x}$

Example: If $\cos x = -\dfrac{3}{5}$, x lies in the third quadrant, find the values of other five trigonometric functions.

Solution: Since $\cos x = -\dfrac{3}{5}$, we have $\sec x = \dfrac{1}{\cos x} = -\dfrac{5}{3}$

Now

$$\sin^2 x + \cos^2 x = 1 \Rightarrow \sin^2 x = 1 - \cos^2 x$$

$$\sin^2 x = 1 - \frac{9}{25} = \frac{16}{25} \Rightarrow \sin x = \pm\frac{4}{5}$$

Since x lies in the third quadrant, $\sin x$ in negative. Therefore $\sin x = -\dfrac{4}{5}$

Therefore $\operatorname{cosec} x = \dfrac{1}{\sin x} = -\dfrac{5}{4}$

Further, we have $\tan x = \dfrac{\sin x}{\cos x} = \dfrac{4}{3}$ and $\cot x = \dfrac{1}{\tan x} = \dfrac{3}{4}$

Example: Find the value of $\cos(-1710^0)$

Solution: We know that values of $\cos x$ repeats after an interval of 2π or 360^0.

Therefore, $\cos(-1710^0) = \cos(-1710^0 + 5\times 360^0) = \cos 90^0 = 0$

Example: Prove that $3\sin\dfrac{\pi}{6}\sec\dfrac{\pi}{3} - 4\sin\dfrac{5\pi}{6}\cot\dfrac{\pi}{4} = 1$

Solution: We have, $\text{LHS} = 3\sin\dfrac{\pi}{6}\sec\dfrac{\pi}{3} - 4\sin\dfrac{5\pi}{6}\cot\dfrac{\pi}{4}$

$$= 3\times\frac{1}{2}\times 2 - 4\sin\left(\pi - \frac{\pi}{6}\right)\times 1 = 3 - 4\sin\frac{\pi}{6}$$

$$= 3 - 4\times\frac{1}{2} = 1 = \text{RHS}$$

Multiple and Submultiples Angles:

- $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta = \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$
- $\sin 2\theta = 2\sin\theta\cos\theta = \dfrac{2\tan\theta}{1+\tan^2\theta}$
- $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$
- $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$
- $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

Transformation from Product to Sum or Difference of Two Angles or Vice-Versa:

- $\sin x + \sin y = 2\sin\left(\dfrac{x+y}{2}\right)\cos\left(\dfrac{x-y}{2}\right)$
- $\sin x - \sin y = 2\sin\left(\dfrac{x-y}{2}\right)\cos\left(\dfrac{x+y}{2}\right)$
- $\cos x + \cos y = 2\cos\left(\dfrac{x+y}{2}\right)\cos\left(\dfrac{x-y}{2}\right)$
- $\cos x - \cos y = 2\sin\left(\dfrac{x+y}{2}\right)\sin\left(\dfrac{y-x}{2}\right)$
- $2\sin x\cos y = \sin(x+y) + \sin(x-y)$
- $2\cos x\sin y = \sin(x+y) - \sin(x-y)$
- $2\cos x\cos y = \cos(x+y) + \cos(x-y)$
- $2\sin x\sin y = \cos(x-y) - \cos(x+y)$
- $\tan 3x = \dfrac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$

Example: Show that $\tan 3x - \tan 2x - \tan x = \tan 3x \tan 2x \tan x$

Solution: $\tan 3x = \tan(2x + x)$

$$\Rightarrow \tan 3x = \frac{\tan 2x + \tan x}{1 - \tan 2x \tan x}$$

$$\Rightarrow \tan 3x(1 - \tan 2x \tan x) = \tan 2x + \tan x$$

$$\Rightarrow \tan 3x - \tan 3x \tan 2x \tan x = \tan 2x + \tan x$$

$$\Rightarrow \tan 3x - \tan 2x - \tan x = \tan 3x \tan 2x \tan x$$

Example: Prove that $\dfrac{\cos 7x+\cos 5x}{\sin 7x-\sin 5x}=\cot x$

Answer:

$$\text{LHS}=\frac{\cos 7x+\cos 5x}{\sin 7x-\sin 5x}=\frac{2\cos\left(\frac{7x+5x}{2}\right)\cos\left(\frac{7x-5x}{2}\right)}{2\cos\left(\frac{7x+5x}{2}\right)\sin\left(\frac{7x-5x}{2}\right)}=\frac{\cos x}{\sin x}=\cot x=\text{RHS}$$

Example: Prove that $\cos 2x\cos\dfrac{x}{2}-\cos 3x\cos\dfrac{9x}{2}=\sin 5x\sin\dfrac{5x}{2}$

Solution: We have

$$\text{LHS}=\cos 2x\cos\frac{x}{2}-\cos 3x\cos\frac{9x}{2}$$

$$=\frac{2\cos 2x\cos\frac{x}{2}}{2}-\frac{2\cos\frac{9x}{2}\cos 3x}{2}$$

$$=\frac{1}{2}\left[\cos\left(2x+\frac{x}{2}\right)+\cos\left(2x-\frac{x}{2}\right)-\cos\left(\frac{9x}{2}+3x\right)-\cos\left(\frac{9x}{2}-3x\right)\right]$$

$$=\frac{1}{2}\left[\cos\frac{5x}{2}+\cos\frac{3x}{2}-\cos\frac{15x}{2}-\cos\frac{3x}{2}\right]$$

$$=\frac{1}{2}\left[\cos\frac{5x}{2}-\cos\frac{15x}{2}\right]=\frac{1}{2}\left[-2\sin\left\{\frac{\frac{5x}{2}+\frac{15x}{2}}{2}\right\}\sin\left\{\frac{\frac{5x}{2}-\frac{15x}{2}}{2}\right\}\right]$$

$$=-\sin 5x\sin\left(-\frac{5x}{2}\right)=\sin 5x\sin\frac{5x}{2}=\text{RHS}$$

Example: Find the value of $\tan\dfrac{\pi}{8}$

Solution: Let $x=\dfrac{\pi}{8}$, Then $2x=\dfrac{\pi}{4}$

$$\Rightarrow\ \tan 2x=\frac{2\tan x}{1-\tan^2 x}\Rightarrow\tan\frac{\pi}{4}=\frac{2\tan\frac{\pi}{8}}{1-\tan^2\frac{\pi}{8}}$$

$$\Rightarrow y=\tan\frac{\pi}{8},\text{ then }\ 1=\frac{2y}{1-y^2}\Rightarrow y^2+2y-1=0$$

Therefore $\quad y=\dfrac{-2\pm 2\sqrt{2}}{2}=-1\pm\sqrt{2}$

Since $\dfrac{\pi}{8}$ lies in the first quadrant, $y=\tan\dfrac{\pi}{8}$ is positive

Hence, $\tan\dfrac{\pi}{8}=\sqrt{2}-1$

Source: NCERT, www.google.com

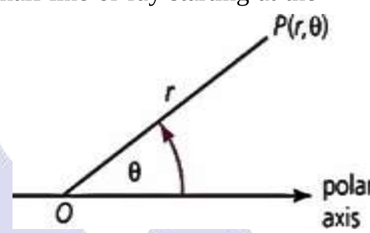
UNIT-5: STRAIGHT LINES

Cartesian and Polar Coordinate:

- In the Cartesian plane, the horizontal line is called the *x*-axis and the vertical line is called the *y*-axis.
- The coordinate axes divide the plane into four parts called quadrants.
- The point of intersection of the axes is called the origin.
- Abscissa or the *x*-coordinate of a point is its distance from the y-axis and the ordinate or the *y*-coordinate is its distance from the *x*-axis,
- (x, y) are called the coordinates of the point whose abscissa is *x* and the ordinate is *y*,
- Coordinates of a point on the *x*-axis are of the form $(x, 0)$ and that of the point on the *y*-axis is of the form $(0, y)$.
- The coordinates of the origin are (0, 0).
- Signs of the coordinates of a point in the first quadrant are (+, +), in the second quadrant (–, +), in the third quadrant (–, –) and in the fourth quadrant (+, –).

Polar Co-ordinates:

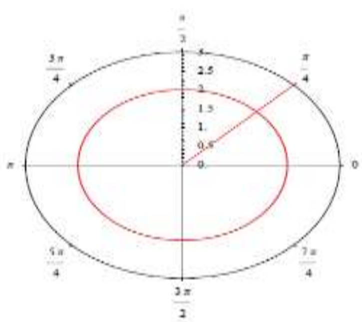
A polar coordinate system, gives the co-ordinates of a point with reference to a point *O* and a half line or ray starting at the point *O*. We will look at polar coordinates for points in the *xy*-plane, using the origin (0, 0) and the positive *x*-axis for reference.



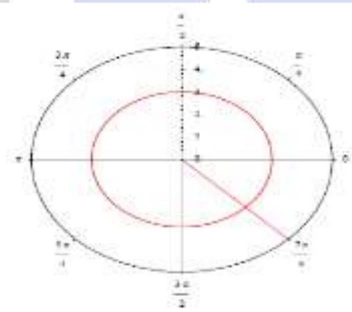
A point *P* in the plane, has polar coordinate (r, θ), where r is the distance of the point from the origin and θ is the angle that the ray $|OP|$ makes with the positive *x*-axis.

Example: Plot the points whose polar coordinates are given by $\left(2, \frac{\pi}{4}\right)$, $\left(3, \frac{7\pi}{4}\right)$

A: $\left(2, \frac{\pi}{4}\right)$



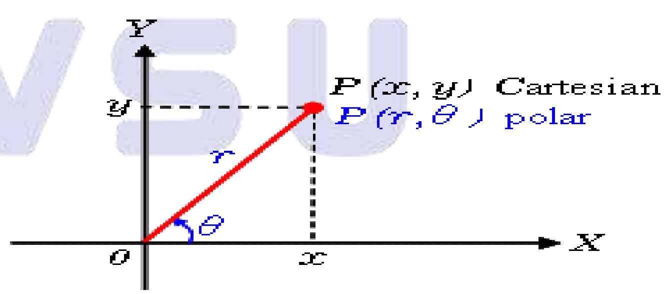
B: $\left(3, \frac{7\pi}{4}\right)$



- The representation of a point in polar coordinate is not unique.

Polar to Cartesian Coordinates and Vice-Versa Conversion:

$$x = r\cos\theta$$
$$y = r\sin\theta$$
$$\text{where, } r = \sqrt{x^2 + y^2}$$
$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$



Source: NCERT, www.google.com

Example: To convert $(1,\ 1)$ to polar coordinates.

Answer: Here,

$$x = 1, \quad y = 1$$

$$\text{Formula:} \quad r = \sqrt{x^2 + y^2} = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{1}{1}\right) = \tan^{-1}(1) = \frac{\pi}{4}$$

So the required polar coordinate = $(r,\ \theta) = \left(\sqrt{2},\ \frac{\pi}{4}\right)$

Example: To convert $\left(3, -\frac{\pi}{3}\right)$ in Cartesian coordinates.

Answer: Here,

$$r = 3, \quad \theta = -\frac{\pi}{3}$$

$$\text{Formula:} \quad x = r\cos\theta, \qquad y = r\sin\theta$$

$$x = 3\cos\left(-\frac{\pi}{3}\right), y = 3\sin\left(-\frac{\pi}{3}\right)$$

$$x = 3 \times \left(\frac{1}{2}\right), \qquad y = 3\left(-\frac{\sqrt{3}}{2}\right)$$

$$x = \frac{3}{2}, \quad y = -\frac{3\sqrt{3}}{2} \qquad \Rightarrow (x,\ y) = \left(\frac{3}{2}, -\frac{3\sqrt{3}}{2}\right)$$

- Distance between the points P (x_1, y_1) and Q (x_2, y_2):

$$\text{PQ} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example: Distance between $(4,6)$ and $(0,3)$ is $= \sqrt{(0-4)^2 + (3-6)^2} = \sqrt{16+9} = 5$ units

- The coordinates of the mid-point of the line segment joining the points (x_1, y_1) and (x_2, y_2) are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
- Area of triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) is:

$$A = \frac{1}{2} | x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) |$$

Example: The area of the triangle, whose vertices are $(0,0)$, $(1,0)$ and $(0,1)$ is

$$A = \frac{1}{2} | x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) |$$

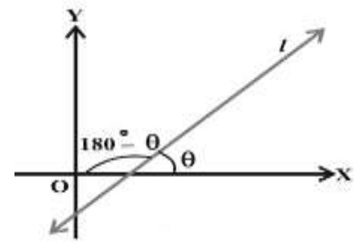
$$A = \frac{1}{2} | 0(0-1) + 1(1-0) + 0(0-0) |$$

$$A = \frac{1}{2} \text{ units}$$

- If the area of the triangle *ABC* is zero, then three points *A*, *B* and *C* lie on a line, *i.e.* they are collinear.

Source: NCERT, www.google.com

Slope of a line: If θ is the inclination of a line, then $\tan\theta$ is called the *slope or gradient* of the line. The slope of a line whose inclination is 90^0 not defined. The slope of a line is denoted by *m*. Thus $m = \tan\theta,\ \theta \neq 90^0$.
It may be observed that the slope of x-axis is zero and slope of y-axis is not defined.



- **Slope of line passing through** $(x_1, y_1), (x_2, y_2)$

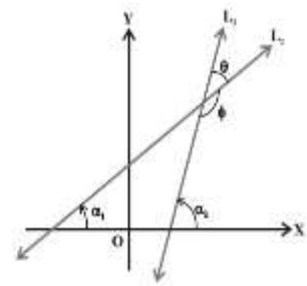
Formula: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Example: To find the slope of line passing through points $(3,-2)$ and $(7,-2)$

Answer: Here, $(x_1, y_1) = (3,-2),\ (x_2, y_2) = (7,-2)$

$$\text{Formula} \quad m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-(-2)}{7-3} = 0$$

- **Angle between two lines:** Let L_1 and L_2 be two non-vertical lines with slope m_1 and m_2 respectively. Let α_1 and α_2 are the inclination of lines L_1 and L_2.



Then $m_1 = \tan\alpha_1,\ m_2 = \tan\alpha_2$

Let θ be the angle of intersection then, $\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$

Note: 1. Two lines are parallel if $m_1 = m_2$

2. Two lines are perpendicular if $m_1 m_2 = -1$

Example: If the angle between two lines is $\dfrac{\pi}{4}$ and slope of the lines is $\dfrac{1}{2}$, find the slope of the other line.

Answer:

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

Here, $m_1 = \dfrac{1}{2},\ m_2 = m$ and $\theta = \dfrac{\pi}{4}$

$$\Rightarrow \tan\frac{\pi}{4} = \left|\frac{\frac{1}{2} - m}{1 + \frac{1}{2}m}\right| \quad \Rightarrow 1 = \left|\frac{\frac{1}{2} - m}{1 + \frac{1}{2}m}\right|$$

$$\Rightarrow 1 = \frac{\frac{1}{2} - m}{1 + \frac{1}{2}m} \quad or \quad \frac{\frac{1}{2} - m}{1 + \frac{1}{2}m} = -1$$

$$\Rightarrow m = -\frac{1}{3} \quad or \quad m = 3$$

Source: NCERT, www.google.com

Different Forms of a Straight Line:

Point Slope Form: Equation of line passing through point (x_0, y_0) and slope m is given by:

$$y - y_0 = m(x - x_0)$$

Example: To find the equation of line through point $(-2,3)$ with slope -4

Answer:

$$\text{Given } (x_0, y_0) = (-2,3) \text{ and } m = -4$$
$$\text{Formula } y - y_0 = m(x - x_0)$$
$$\Rightarrow y - 3 = -4(x + 2)$$
$$\Rightarrow 4x + y + 5 = 0$$

Two-Point Form: Equation of line passing through points (x_1, y_1) and (x_2, y_2) is given by:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Example: To find the equation of line passing through the points $(1,-1)$ and $(3,5)$

Answer:

$$\text{Given } (x_1, y_1) = (1,-1) \text{ and } (x_2, y_2) = (3,5)$$
$$\text{Formula } y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$
$$\Rightarrow y - (-1) = \frac{5 - (-1)}{3 - 1}(x - 1)$$
$$\Rightarrow -3x + y + 4 = 0$$

Slope- Intercept Form: Equation of line with slope m and intercept c on *y*-axis is given by:

$$y = mx + c$$

Example: To find the equation of line with slope -1 and intercept of 2 on *y*-axis.

Answer:

$$\text{Given } m = -1 \; and \; c = 2$$
$$\text{Formula } y = mx + c$$
$$\Rightarrow y = -x + 2$$
$$\Rightarrow x + y - 2 = 0$$

Intercept Form: Equation of line making intercepts of a and b on *x*-axis and *y*-axis respectively is given by:

$$\frac{x}{a} + \frac{y}{b} = 1$$

Example: To find the equation of line, which makes intercepts -2 and 3 on *x*-axis and *y*-axis respectively.

Answer: Given $a=-2$ and b = 3

Formula $\frac{x}{a}+\frac{y}{b}=1 \Rightarrow \frac{x}{-2}+\frac{y}{3}=1 \Rightarrow 3x-2y+6=0$

General Equation of a Line: General equation of line is given by: $Ax+By+C=0$, Where A,B are not zero. The slope of equation is $m=-\frac{A}{B}$

Example: Equation of line is $3x-4y+12=0$. To find its (I) Slope and (II) *x*- and *y*-intercepts.

Answer: (I) Given equation of line: $3x-4y+12=0$

$$\Rightarrow 4y=3x+12 \Rightarrow y=\left(\frac{3}{4}\right)x+3$$

Comparing with slope-intercept form $y=mx+c$, we have slope of the line $m=\frac{3}{4}$

(II) Given equation of line:

$$3x-4y+12=0$$

$$\Rightarrow 3x-4y=-12 \Rightarrow \left(\frac{3x}{-12}\right)+\left(\frac{-4y}{-12}\right)=0 \Rightarrow \frac{x}{\left(\frac{-12}{3}\right)}+\frac{y}{\left(\frac{-12}{-4}\right)}=0$$

$$\Rightarrow \frac{x}{(-4)}+\frac{y}{(3)}=0$$

Comparing with intercepts form of line $\frac{x}{a}+\frac{y}{b}=1$, we have $a=-4,\ b=3$

Distance of a Point from a Line: The distance of a point from a given line is the length of the perpendicular drawn from the point to the line. Let equation of line is: $Ax+By+C=0$. Its distance from the point $P(x_0,y_0)$ is d.

Then $d=\frac{|Ax_0+By_0+C|}{\sqrt{A^2+B^2}}$

Example: To find the distance of the point $(1,-2)$ from the line $4x-3y+9=0$.

Answer: Given point $(x_1,y_1)=(1,-2)$ and line $4x-3y+9=0$

Comparing with general equation of line $Ax+By+C=0$, we have: $A=4, B=-3, C=9$

$$d=\frac{|Ax_0+By_0+C|}{\sqrt{A^2+B^2}}$$

$$\Rightarrow d=\frac{|4\times1-3\times(-2)+9|}{\sqrt{4^2+(-3)^2}} \Rightarrow d=\frac{|4+6+9|}{\sqrt{16+9}} \Rightarrow d=\frac{15}{5} \Rightarrow d=3\text{units}$$

Source: NCERT, www.google.com